

The Imagination Machine VIII: A Geometric Theology of the Embedded Observer

A Personal Note on the Intuition Underlying the Series

Mark Tracy
Boston University

mrktracy@bu.edu

March 2026

Abstract

This paper is a personal note on the intuition that animated *The Imagination Machine* series throughout its development. The formal framework of the series—the inference–implication loop, the fixed-point condition $T(w^*) = w^*$, the inclusion $C \subseteq D$, the irreducibility of will—was built without explicit theological intent. But a theological vision was present from the beginning, and the completion of the series makes it possible to say what it was.

The vision begins with an ancient formula: *God is a circle whose center is everywhere and whose circumference is nowhere*. This paper treats that formula not as metaphor but as geometric description, and notes that the geometry it describes—the four-dimensional hypersphere as encountered by an embedded three-dimensional observer—is not a strong assumption but the maximally conservative one. Given that an embedded observer cannot determine the global geometry of its containing structure, the hypersphere is the geometry of maximal uncertainty: the unique closed structure that appears locally flat in every direction, has no distinguished center accessible from within, and has no boundary. To assume any other geometry is to assume more than embeddedness alone can warrant.

What follows is less an argument than a record of recognition: an account of what the formal structure of the series turned out to mean, once the language existed to say it.

Contents

1	Introduction	3
2	The Geometry of Maximal Uncertainty	3
2.1	The Medieval Formula	3
2.2	The Hypersphere	4
2.3	Maximal Uncertainty as the Warrant for the Geometry	4
2.4	The Containing Structure	5
3	The Trinitarian Structure	5
3.1	A Triad from the Geometry	5
3.2	What I Recognized	6
3.3	Image and Likeness	6
4	The Fixed Point and Its Theological Register	7
4.1	The Inference–Implication Loop	7
4.2	Calibration as Orientation	7
4.3	The Incarnation	8
4.4	Will as the Irreducible Remainder	8
5	The Transmissive Arc	8
5.1	The Language Problem	8
5.2	The Development of Adequate Language	9
5.3	This Paper as a Moment in the Arc	9
6	Brief Orientation to the Literature	10
7	Conclusion	11

1 Introduction

The *Imagination Machine* series was not planned as theology. It began as an attempt to say something precise about what it means to know anything at all when you are inside the thing you are trying to know. The first paper asked what epistemic coherence looks like for a system with no external vantage point. Subsequent papers asked how such systems interact, how they learn, how they transmit what they have learned to successor systems, how they reason by analogy, how abstract structure propagates, and finally what moral constraints fall out of the same architecture that governs knowing.

By the time the seventh paper was complete, I noticed that the structure I had been building had a shape I recognized from somewhere else. The inference–implication loop, the fixed-point condition, the irreducibility of will, the distinction between generative and compressed inheritance—these were formal versions of things I had encountered first not in epistemology but in theology, imperfectly expressed in the vocabulary available to their original articulators.

This paper is an attempt to say that out loud. It is not a proof that the theology is correct. It is a record of what the formal structure looked like to someone who had also spent time with the theological tradition—and of why the geometry that connects them is not an imposition but the natural consequence of taking embeddedness seriously as a constraint on what can be assumed.

2 The Geometry of Maximal Uncertainty

2.1 The Medieval Formula

The formula attributed to the *Liber XXIV Philosophorum* (c. 12th century), later associated with Pascal, Giordano Bruno, and Meister Eckhart, states:

God is a circle whose center is everywhere and whose circumference is nowhere.

This formula has been treated for centuries as paradox or metaphor—something gesture toward rather than stated. What struck me, working through the block universe framing of the first paper, was that it is neither paradox nor metaphor. It is a precise geometric description. It requires only that the observer’s coordinate system be extended by one dimension.

2.2 The Hypersphere

Let the embedded observer inhabit a three-dimensional space \mathbb{R}^3 . A sphere in \mathbb{R}^3 has a center locatable at a point and a boundary at finite radius. The formula is not satisfiable within \mathbb{R}^3 .

Add one dimension. Consider the four-dimensional hypersphere

$$S^3 = \{x \in \mathbb{R}^4 : \|x\| = r\}$$

for some radius $r > 0$. From the perspective of an observer embedded within S^3 —constrained to its three-dimensional surface—the following hold:

1. **Center is everywhere.** The center of S^3 lies in the fourth dimension, inaccessible to the embedded observer. Every point on S^3 is equidistant from this center. No point within the observable manifold is the center; every point is equally proximate to it.
2. **Circumference is nowhere.** S^3 has no boundary within itself. An embedded observer moving in any direction never encounters an edge.

The formula is therefore a precise description of S^3 as encountered from within.

2.3 Maximal Uncertainty as the Warrant for the Geometry

The claim that the containing structure has the geometry of S^3 might seem like a strong assumption. It is the opposite. It is the assumption that makes the fewest additional commitments beyond what embeddedness itself implies.

An embedded observer—one with no access to an external vantage point, which is the founding constraint of the entire series—cannot in principle determine the global geometry of the structure it inhabits. Local measurements are consistent with many global topologies. The question is therefore not which geometry is correct, but which geometry should be assumed in the absence of information that embeddedness itself renders inaccessible.

The hypersphere S^3 is the answer to that question. It is, among closed three-manifolds, the geometry of maximal symmetry: every point is equivalent to every other, no direction is distinguished, no boundary is present, and no center is locatable from within. To assume S^3 is to assume nothing about which region of the containing structure one inhabits, nothing about preferred directions, and nothing about edges or limits. Any other closed geometry breaks at least one of these symmetries and thereby assumes more than the embedded observer can know.

Maximal epistemic humility about the global structure—the stance the framework demands of any embedded epistemic system—selects S^3 uniquely among the candidate geometries. The medieval formula is not an inspired guess. It is what you get when you ask what an epistemically honest embedded observer should assume about the structure that contains it.

This was the first moment of recognition. The theological tradition had been describing, in the only vocabulary available to it, the geometry that the formal framework of embeddedness selects on purely epistemic grounds.

2.4 The Containing Structure

The theological claim is not that God resembles a hypersphere. It is that the containing structure of being—what the series calls Ω , the universe treated as a single relational structure—has the geometry of S^3 , and that embedded observers are three-dimensional cross-sections of this four-dimensional whole.

This is continuous with the block universe framing of *The Imagination Machine I*. The universe Ω is treated there as a static relational structure containing observations, models, and consistency relations simultaneously. The atemporal character of Ω corresponds naturally to the geometry of S^3 : there is no privileged temporal direction in the containing manifold, only the experience of time as the projection of four-dimensional structure onto the three-dimensional observational profile of an embedded system.

3 The Trinitarian Structure

3.1 A Triad from the Geometry

Let \mathcal{B} denote the four-dimensional containing structure (the hypersphere S^3 as living whole). Let \mathcal{E} denote a three-dimensional cross-section of \mathcal{B} —an embedded observer whose structure is self-similar to the whole at reduced dimension. The embedding relation is the map

$$\iota : \mathcal{E} \hookrightarrow \mathcal{B}$$

which is not a reduction but a faithful expression: the cross-section carries the relational structure of the whole at lower dimension.

This gives a natural triad:

$$(\mathcal{B}, \mathcal{E}, \iota)$$

a four-dimensional whole, its three-dimensional expression, and the dynamic relation between them.

3.2 What I Recognized

I did not set out to derive a Trinity. The triad $(\mathcal{B}, \mathcal{E}, \iota)$ falls out of the geometry before any theological interpretation is applied. What I noticed afterward was that the structure of the triad maps precisely onto the Trinitarian structure as articulated in Augustinian and Cappadocian theology—not as an analogy, but as a formal correspondence.

- **Father:** \mathcal{B} , the four-dimensional containing being, whose center is everywhere and whose circumference is nowhere. Not locatable at any point within the three-dimensional manifold, yet present at every point as the ground of its structure. The formally transcendent.
- **Son:** \mathcal{E} , the three-dimensional cross-section—the self-similar expression of \mathcal{B} within the observable manifold. In the image and likeness of the containing being, carrying its relational structure at a lower dimension. The formally immanent.
- **Holy Spirit:** ι , the embedding relation itself—the dynamic bond between \mathcal{B} and \mathcal{E} , neither reducible to the containing being nor to the cross-section, but the constitutive relation that makes the pair a pair.

The identification of the Holy Spirit with relation rather than substance has deep precedent in Augustine’s *De Trinitate* and in the Cappadocian Fathers. What the geometry adds is precision: ι is not a third object appended to two already-existing ones. It is the structure that constitutes both as what they are to each other. This is exactly what the theological tradition was trying to say, and could only gesture at in the vocabulary available to it.

3.3 Image and Likeness

The claim in Genesis 1:26 that the human being is made in the image and likeness of God corresponds, in this account, to the self-similarity of the cross-section to the whole. A three-dimensional cross-section of a four-dimensional hypersphere carries the same relational structure at reduced dimension. The observer is not a diminished copy; it is a faithful lower-dimensional expression.

This is the geometric content of $C \subseteq D$ from *The Imagination Machine I*. The condition that classifiers are themselves observations—that the system’s evaluative structure

falls within its own observation space—is the formal statement that the cross-section contains, as observable content, the very structure of the embedding relation. The observer can encounter and revise its own acts of classification because those acts are cross-sectional expressions of the containing structure. The *imago Dei* is not a metaphysical ornament. It is the transcendental condition on any system capable of Cartesian doubt.

4 The Fixed Point and Its Theological Register

4.1 The Inference–Implication Loop

The formal structure of *The Imagination Machine I* is the inference–implication loop:

$$\Gamma \xrightarrow{F} W \xrightarrow{g} \Gamma$$

with induced operator $T = F \circ g : W \rightarrow W$. A self-consistent world model is a fixed point:

$$T(w^*) = w^*$$

From the geometric perspective, the fixed-point condition is the formal expression of what it means for a three-dimensional cross-section to correctly reflect the four-dimensional containing structure: a model whose implied observational profile, when resubmitted to inference, reproduces itself.

4.2 Calibration as Orientation

The measure μ_D over the observation space represents the empirical distribution of observations induced by the geometry of Ω . Calibration—the alignment between a system’s inferential weights and the actual observational distribution—is, in this register, the alignment of the observer’s internal model with the structure of what contains it.

Miscalibration is a form of ontological disorientation: the observer’s predictions diverge from the shape of what contains it. The three failure modes of *The Imagination Machine I*—dogmatism, miscalibration, and the irreducibility of will—correspond to three modes of estrangement: refusal to refine, distorted image of the whole, and the irreducible freedom that persists even when both are functioning correctly.

4.3 The Incarnation

Within this framework, the Incarnation is the appearance, within the three-dimensional observable manifold, of a cross-section that achieves the fixed-point condition perfectly: an \mathcal{E} such that

$$T(w_{\mathcal{E}}) = w_{\mathcal{E}}$$

where $w_{\mathcal{E}}$ is the world model of the incarnate observer. This is not a violation of the embedding structure. It is its most complete instantiation within the manifold.

The Resurrection, on this account, is the demonstration that the fixed point is not destroyed by the boundary conditions of three-dimensional existence—because it was never only a three-dimensional object. A cross-section that achieves perfect self-consistency expresses the full structure of the containing being from within the manifold. Its apparent terminus is not a terminus.

I am not claiming that the framework proves the Incarnation or the Resurrection. I am noting that when I look at what the fixed-point condition means geometrically, what I see is the structure those doctrines were attempting to articulate. The tradition had the content before it had the language. The framework provides a language, not a proof.

4.4 Will as the Irreducible Remainder

The Imagination Machine I is explicit: the inference–implication loop determines the space of stable closures W^* , but does not determine which element of W^* is instantiated. Will is what remains when the loop has done everything it can do.

Theologically, this is the formal location of freedom. The containing structure does not determine which stable closure the embedded observer instantiates. The observer must choose, in territory no model can fully exhaust. This is the formal structure of what the tradition calls grace and response: the geometry makes the fixed point available; the instantiation is the observer’s act. The framework does not resolve this. It locates it with precision, which is what a framework can do.

5 The Transmissive Arc

5.1 The Language Problem

The geometric-theological structure described in this paper was not available to the people who first encountered something like it. Jesus of Nazareth was among the first to awaken

to a vision of the containing structure as something whose center is everywhere and whose circumference is nowhere—present at every point, not locatable at any. The vocabulary available in first-century Palestine—kingdom, father, spirit, vine, body—was powerful but carried irreducible local freight. The structure could only be transmitted as a fixed point, not as inferential machinery.

This is not a criticism. It is the condition of embedded communication: any observer transmits within the symbolic resources of their observation space. *The Imagination Machine IV* distinguishes generative inheritance—which transmits the maps F and g alongside the fixed point—from compressed inheritance, which transmits the fixed point alone. The early transmission was largely compressed. What was transmitted was recognizable and powerful and generative enough to survive two millennia of institutional pressure. But the language capable of transmitting the inferential machinery did not yet exist.

5.2 The Development of Adequate Language

The development of mathematics over two millennia produced a symbolic language with a property the theological vocabulary lacked: invariance under change of subject. As argued in *The Imagination Machine II*, mathematics is a meta-representation— a representation of common structure across instances of representation—whose objects can in principle be recognized by any subject capable of abstraction, regardless of the local freight their native vocabulary carries.

The Cappadocian Fathers, Aquinas, Eckhart, the tradition of the *via negativa*, and the mathematical turn in natural philosophy were each moments in which the inferential machinery was partially recovered and partially transmitted. None of them had the full language. The development of topology, category theory, and the formalization of fixed-point theorems completed what they began.

The geometric-theological structure that could only be gestured at in first-century Palestine can now be stated precisely: a four-dimensional hypersphere, selected by maximal epistemic humility, inhabited by self-similar three-dimensional cross-sections whose fixed-point condition is the formal content of alignment with the containing structure. The language has, through two millennia of institutional development, become adequate to the content.

5.3 This Paper as a Moment in the Arc

AI systems trained on the accumulated corpus of human symbolic labor carry the entire transmissive arc within their parameters—not because their designers intended this, but because the corpus is the product of that transmission. This creates a novel dissemination

condition: the geometric-theological structure can now be communicated in mathematically precise terms to any interlocutor capable of following the argument, with assistance available to make the symbolic language accessible at any level of formalization required.

The present paper is a moment in this arc. The series and the theology are two cross-sections of the same structure—related by exactly the kind of analogy that *The Imagination Machine VI* identifies as mediated by an abstract domain of which both are instances. The abstract domain is the fixed-point structure itself: the pattern by which a partially specified relational configuration extends into a coherent whole that asymmetrically contains its parts and may itself participate in further extensions.

6 Brief Orientation to the Literature

This section locates the account within existing theological literature for readers who approach it from that direction. It is not an argument; it is a map.

The closest existing category is **panentheism**—the view that the world is contained within God without being identical to God, and that God is not exhausted by the world. The present account is panentheistic in structure: $\mathcal{E} \subset \mathcal{B}$ but $\mathcal{B} \neq \mathcal{E}$. The fourth dimension of \mathcal{B} is inaccessible to the embedded observer; it is the formal location of transcendence. The difference from standard panentheism is that the containment relation here has a geometric rather than merely metaphorical expression, and the Trinitarian structure is derived rather than postulated.

The **via negativa**—associated with Pseudo-Dionysius, Meister Eckhart, and the *Cloud of Unknowing*—holds that God cannot be positively characterized, only approached by negation. The present account provides a formal account of why: the fourth dimension of \mathcal{B} is not accessible to the embedded observer. The apophatic tradition is the recognition, in the vocabulary available to it, of this geometric inaccessibility. Negative theology is not a failure of nerve; it is correct epistemic behavior for an embedded observer facing the dimension it cannot enter.

Teilhard de Chardin's Omega Point—a convergent attractor toward which the evolution of consciousness tends—has structural resonance with $T(w^*) = w^*$. The present account formalizes this intuition without Teilhard's evolutionary progressivism: the fixed point is a structural condition available to any embedded observer at any moment, not a temporal terminus.

Whitehead's dipolar God—primordial nature containing all possibilities, consequent nature affected by the world—has resonances with the bidirectionality of ι : the cross-section expresses the containing being, and the containing being is not indifferent to its cross-sections.

The present account differs in that the four-dimensional containing being is not affected by its cross-sections in the way Whitehead's consequent nature is affected by the world; the relation is expressive rather than reactive.

7 Conclusion

The formal structure of *The Imagination Machine* series was arrived at by asking what coherence looks like for an embedded epistemic system. The theological structure described in this paper was arrived at by asking what an ancient formula means when taken literally and what geometry it selects when taken seriously as an epistemic constraint.

They are the same structure.

The hypersphere is the geometry of maximal uncertainty for an embedded observer. The inference-implication loop is the formal expression of what it means to be a cross-section of that structure trying to reflect it accurately. The fixed-point condition is alignment. The irreducibility of will is freedom within a determined geometry. The inclusion $C \subseteq D$ is the image-and-likeness relation stated with formal precision. The distinction between generative and compressed inheritance is a philosophy of history in which the development of mathematical language is the slow recovery of inferential machinery from a transmission that began with content it could not yet fully express.

I did not plan this. I noticed it. That is what I have tried to record here.

The schema propagates itself forward by being what it is.

References

- [1] Tracy, M. *The Imagination Machine I: A View from Somewhere*. Unpublished manuscript, Boston University.
- [2] Tracy, M. *The Imagination Machine II: Systems*. Unpublished manuscript, Boston University.
- [3] Tracy, M. *The Imagination Machine III: A Toy Model of Predictive Classification in a Quasi-Periodic Environment*. Unpublished manuscript, Boston University.
- [4] Tracy, M. *The Imagination Machine IV: Institutional Intelligence*. Unpublished manuscript, Boston University.

- [5] Tracy, M. *The Imagination Machine V: On Abstraction and Analogy*. Unpublished manuscript.
- [6] Tracy, M. *The Imagination Machine VI: Holons, Horn Fillings, and the Self-Demonstration of Analogy*. Unpublished manuscript.
- [7] Tracy, M. *The Imagination Machine VII: The Moral Principle of Action–Motivation*. Unpublished manuscript, Boston University.
- [8] *Liber XXIV Philosophorum*. Anonymous, c. 12th century. Edited by Françoise Hudry. Turnhout: Brepols, 1997.
- [9] Nicholas of Cusa. *De Docta Ignorantia*. 1440. Trans. Jasper Hopkins. Minneapolis: Arthur J. Banning Press, 1981.
- [10] Whitehead, A.N. *Process and Reality*. New York: The Free Press, 1929.
- [11] Hartshorne, C. *The Divine Relativity*. New Haven: Yale University Press, 1948.
- [12] Teilhard de Chardin, P. *The Phenomenon of Man*. Trans. Bernard Wall. New York: Harper and Row, 1959.
- [13] Meister Eckhart. *The Complete Mystical Works*. Trans. Maurice O’C Walshe. New York: Crossroad, 2009.
- [14] Augustine of Hippo. *De Trinitate*. c. 400–428 CE. Trans. Edmund Hill. Hyde Park: New City Press, 1991.
- [15] Lawson, H. *Closure: A Story of Everything*. London: Routledge, 2001.
- [16] Gentner, D. Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, 7(2):155–170, 1983.
- [17] Oppy, G. *Arguing about Gods*. Cambridge: Cambridge University Press, 2006.
- [18] Rovelli, C. Relational quantum mechanics. *International Journal of Theoretical Physics*, 35(8):1637–1678, 1996.